Advanced Quantification of Urban Complexity and Adaptive Capacity: Sub-Fractal Analysis and Spatial Statistics in Izmir

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Abstract
This study evaluates the adaptive capacity of İzmir, Turkey, by applying sub-fractal and difference analysis to assess urban complexity from the 1950s to 2020s. Using principles of fractal geometry (Batty & Longley, 1994), it quantifies the morphological evolution of the city's road network to determine resilience to socio-environmental change. The findings identify a consistent Adaptive Capacity Range (ACR) of 0–0.25, indicating moderate adaptability constrained by structural thresholds (Özdemir, 2021; Erdoğan, 2021). Drawing on statistical mechanics and theoretical physics, the study introduces computational tools for modelling urban expansion (Salingaros, 2010) and offers a transferable framework for resilience assessment (Triantakonstantis, 2012; Jiang & Yin, 2014). This interdisciplinary approach advances understanding of urban complexity and provides a replicable model for analysing adaptive capacity in dynamic city systems.

Keywords: Adaptive Capacity, Sub-Fractal Analysis, Urban Complexity, Urban Planning, Fractal Geometry, Urban Growth

1. Introduction

The dynamics of urban complexity and adaptive capacity occupy a central position in contemporary urban planning theory. These constructs embody the multifaceted interactions between spatial configurations, infrastructural systems, and the socio-economic mechanisms that sustain urban life (Salingaros, 2005). Adaptive capacity, originally conceptualized within ecological and environmental sciences, refers to a system's potential—whether biological, ecological, or social—to adjust to disturbances and changes while preserving its fundamental functions (Salingaros, 2010). Closely tied to the notion of resilience, it denotes a system's ability to reorganize amidst disruptions without forfeiting its core identity and operational integrity.

This concept aligns conceptually with the principles of fractal geometry, which have proven instrumental in interpreting urban form and complexity (Batty & Longley, 1994). Fractal geometry elucidates urban spatial organization and developmental trajectories by revealing scale-invariant and self-similar patterns—allowing complexity to be both measured and visualized (Triantakonstantis, 2012). Situated within this analytical framework, the present study employs fractal methodologies to explore the morphological development of İzmir, Turkey, tracing the evolution of its urban spatiality across decades shaped by both deliberate planning and spontaneous transformation (Özdemir, 2021; Erdoğan, 2021).

As Turkey's third-largest metropolitan area, İzmir offers a compelling case for examining urban adaptability through the lens of sub-fractal analysis (Erdoğan, 2021). This technique dissects the granularity of urban forms across varying scales, thereby illuminating how spatial structure influences responsiveness to socio-economic and environmental perturbations (Jiang & Liu, 2009). In tandem, difference analysis tracks temporal shifts in fractal configurations, providing empirical evidence of the impacts of historical planning initiatives and unforeseen urban developments. Anchored in interdisciplinary theory spanning urban morphology and computational physics, the study utilizes quantitative methods to interrogate the nexus between form and adaptive functionality (Jiang & Claramunt, 2004; Jiang & Yin, 2014).

By integrating these analytical tools, the research articulates actionable insights for urban policy and planning, advocating for data-informed strategies that bolster urban resilience and adaptability (Salingaros, 2005; Özdemir, 2021). Ultimately, the study advances the formulation of standardized methodologies capable of evaluating adaptive capacity across diverse urban settings, contributing to a globally relevant framework for resilience assessment (Batty & Longley, 1994; Triantakonstantis, 2012).

2. Measurement Techniques and Data

Urban complexity constitutes a foundational dimension in evaluating cities' capacity to adapt to evolving socio-spatial and environmental dynamics. Among the most salient methodological tools for investigating this complexity is fractal analysis, which offers a quantitative framework for identifying recursive, self-similar patterns inherent in urban form—patterns that elucidate dynamics of sprawl, density, and morphological transformation (Batty & Longley, 1994; Frankhauser, 1998). Extending this approach, sub-fractal analysis interrogates the internal complexity of specific urban elements, thereby revealing fine-grained heterogeneities that are often obscured at broader scales (Allain & Cloitre, 1991). In the context of İzmir, such an analysis facilitates the exploration of inter-district variability in spatial complexity, closely tied to differential capacities for urban adaptation.

Difference analysis complements this by capturing temporal shifts in urban complexity through comparative evaluations of fractal dimensions across multiple historical junctures. This technique effectively foregrounds the impact of strategic planning decisions and unanticipated perturbations on the urban fabric, rendering visible structural discontinuities in density and sprawl patterns (Batty & Longley, 1994). Applied to İzmir, it enables a retrospective assessment of how specific interventions have altered resilience trajectories. To operationalize an assessment of İzmir's adaptive capacity, sub-fractal analysis is employed to measure the evolution of the city's road network across temporally distinct phases, as indicated by fluctuations in fractal dimensionality (Salingaros, 2010; Jiang & Yin, 2014). This methodological lens serves to index the structural plasticity of the urban grid by mapping its complexity dynamics over time. The empirical basis for this analysis comprises a series of high-resolution cartographic datasets from the General Directorate of Maps, each rendered at a 1:25,000 scale and covering İzmir's metropolitan territory during four historical intervals: 1958–1964, 1974–1980, 1996–2000, and 2012–2020. These maps, standardized in cartographic design and digitally harmonized via NetCAD software, provide a coherent temporal sequence for evaluating the morphogenetic evolution of the urban road network. Subsequent image processing and complexity quantification are conducted using Frac_Lac software, with parameters set at a 1%-pixel ratio and an R^2 coefficient consistently exceeding 0.999, ensuring statistical reliability. The spatial scope of the study is carefully delineated to coincide with İzmir's official metropolitan jurisdiction during each examined period, particularly aligning with the master planning boundaries that underpin the respective base maps of the urban transportation network. Within these demarcated regions, a corpus of sixteen geospatially referenced maps is analyzed to estimate the city's adaptive resilience. The master plans informing this analysis include:

- The 1952 competition proposal by Aru, **Özdeş**, and Canpolat, revised in 1955.
- The 1972 Metropolitan Planning Office's Plan, authorized in 1973 and revised in 1978.
- The 1989 1/50,000 scale plan by the Metropolitan Municipality.
- The 2012 1/25,000 scale plan by the Metropolitan Municipality of İzmir.

3. Methodology

3.1. Sub-fractal and Sub-fractal Difference Analysis

Sub-fractal analysis constitutes a refined methodological approach for interrogating the spatial intricacies of urban form by isolating and quantifying the geometric complexity of discrete urban components—such as transportation infrastructures or neighbourhood morphologies. Rooted in the theoretical underpinnings of fractal geometry, this technique elucidates scale-invariant, self-replicating patterns that typify urban spatial organization (Frankhauser, 1998; Chen, 2021).

Operationalized through a mesh-grid framework, sub-fractal analysis determines the minimal count of non-overlapping square boxes, denoted as $N_{(S)}$, necessary to comprehensively enclose a given urban structure at a specified box size S. The fractal dimension D, an index of spatial complexity, is derived from the linear relationship between the natural logarithms of $N_{(S)}$ and S, where the slope of the resulting log-log regression line yields:

$$D = \lim_{\Delta_S \to 0} \left(-\frac{\ln N_S}{\ln S} \right)$$

The regression model is typically expressed as:

$$\ln N_S = A + D \ln \left(\frac{1}{S} \right) + E_S$$

In this regression-based approach, A signifies the intercept, D denotes the estimated fractal dimension, and E_S represents the standard error term associated with the model. The robustness of this method is affirmed by its requirement for a coefficient of determination typically exceeding $R^2 > 0.999$, as demonstrated in the work of Thomas and Frankhauser (2013). For the application in İzmir, the sub-fractal analysis was conducted on the map encompassing the most extensive geographical coverage within the delineated urban planning boundaries. A minimum analytical window of 800 m × 800 m was employed, corresponding to a 1%-pixel resolution, which enabled a high-resolution assessment of spatial complexity across multiple temporal epochs (see Figure 2 for visualization). Building upon this foundation, sub-fractal difference analysis examines temporal shifts in spatial complexity. This technique involves computing the variation in sub-fractal dimension values for each individual grid cell across two temporal snapshots. The change is expressed as:

$$\Delta Sf = Sf_{post} - Sf_{pre}$$

Where Sf_{pre} denotes the sub-fractal value in the pre-plan period and Sf_{post} indicates the corresponding value in the post-plan period, the resulting ΔSf expresses the net change in spatial complexity. This differential value serves as a quantitative indicator of morphological transformation, signalling either an increase or a decrease in urban complexity (Chen, 2021; Thomas & Frankhauser, 2013). The computed ΔSf values are systematically grouped into fixed intervals, typically segmented using a threshold of 0.25, to establish what is referred to as the Adaptive Capacity Range (ACR). The ACR represents the standard interval within which complexity changes are considered typical across the urban landscape. Subsequently, a visual map is generated to spatially represent the variation in ΔSf values, employing a color-coded gradient to differentiate the magnitude and direction of change:

- Cool colors (indicating negative ΔSf values) reflect a reduction in complexity.
- Warm colors (indicating positive ΔSf values) signify an increase in complexity.

This cartographic output (see Figure 3) captures the spatiotemporal dynamics of urban transformation, illuminating how different zones within İzmir react to urban planning interventions. By analysing these fluctuations, the study provides a nuanced assessment of the city's adaptive responses over time, revealing patterns of resilience and structural evolution shaped by specific policy decisions (Sutton et al., 1997; Thomas & Frankhauser, 2013).

3.2. Statistical Analysis of *Adaptive Capacity Range*

Within the framework of urban systems theory, the capacity to adapt to environmental, socio-economic, or institutional perturbations is a foundational determinant of long-term sustainability and functional integrity. The Adaptive Capacity Range (ACR) serves as a diagnostic metric for assessing this resilience. A positive ACR value signifies that the urban system is endowed with sufficient structural and organizational flexibility to withstand and absorb external shocks, thereby maintaining continuity and performance under changing conditions. In contrast, an ACR value that approaches zero or becomes negative would imply an absence of adaptive mechanisms, effectively indicating systemic rigidity and heightened vulnerability to disruption—conditions that could precipitate breakdown or systemic collapse. Such a scenario is antithetical to the principles of urban resilience, which posit that adaptive capacity is an indispensable element in sustaining viable urban environments (Ahern, 2011).

Consequently, by theoretical and practical definition, ***ACR must invariably assume a positive value to meaningfully reflect the presence of adaptability.*** A zero or negative ACR would conceptually denote total incapacity for adjustment—an implausible condition for any operational urban system. Positive ACR values thus ensure the existence of a minimal threshold of resilience embedded within the spatial and functional structure of the city.

The Adaptive Capacity Range (ACR) is formally defined as:

$$\boldsymbol{ACR = [\mu - k\sigma, \mu + k\sigma]} \ \ \textit{where} \ \boldsymbol{\mu - k\sigma > 0}$$

In this formulation, μ represents the arithmetic mean, σ denotes the standard deviation, and k is a coefficient corresponding to a selected confidence interval.

To ensure that the Adaptive Capacity Range (ACR) robustly captures the central tendencies of adaptive behaviour, both the mean and median values from each analysed time period are incorporated. The inclusion of these statistical descriptors is essential: the mean reflects the average tendency across all grid cells, while the median highlights the central point of the data distribution, mitigating the influence of outliers or skewed patterns. This dual-statistic approach allows for a more comprehensive representation of urban adaptability over time (Hu & He, 2018).

Without accounting for both the mean and the median across all measurement intervals, key variations in adaptive capacity may remain undetected, thereby compromising the reliability and interpretive depth of the analysis. ***Therefore, for the ACR to effectively capture the dynamics of urban resilience, it must systematically include the mean and median values derived from each temporal stage of analysis.***

Formally, for each time period ti, the ACR is defined as a function of both:

$$\boldsymbol{ACR(t_i) = [\mu(t_i) - k\sigma(t_i), \mu(t_i) + k\sigma(t_i)]} \ \ \textit{where} \ \boldsymbol{\mu(t_i) - k\sigma(t_i) > 0}$$

In this framework, μ(ti) and i(ti) denote the mean and standard deviation of adaptive capacity at a given time point ti, respectively.

Sub-fractal analysis, by its design, enables the detection of fine-grained spatial variations embedded within the urban morphological structure. To preserve the analytical coherence of the Adaptive Capacity Range (ACR) as a holistic indicator of variability, it is imperative that differences in sub-fractal mean values across time intervals remain bounded within the limits defined by the ACR. This constraint ensures that abrupt anomalies or exaggerated deviations do not compromise the validity of the model, thereby sustaining the consistency and robustness of longitudinal interpretations.

Accordingly, the deviation between sub-fractal means at any two measurement points must not surpass the ACR. This stipulation guarantees that the variability in adaptive behaviour, as indexed by mean sub-fractal values, is effectively encapsulated within the expected bounds of system adaptability. It mitigates the risk of statistical distortion and upholds the integrity of the model's predictive and descriptive capacities. More precisely, to avoid overstating variability, the absolute difference in sub-fractal means must never exceed twice the amplitude (2A) of the ACR. Here, the amplitude A denotes the maximum permissible deviation from the central value (typically the mean) within the ACR. Constraining sub-fractal variability to within ±A ensures symmetrical balance, while enforcing the 2A threshold prevents the emergence of outlier patterns that could misrepresent adaptive trends.

This condition is formally expressed as:

$$2A \geq \left|\mu(t_i) - \mu(t_j)\right|$$

In this context, **A** denotes the amplitude, which defines the extent of variability within the Adaptive Capacity Range (ACR). A narrower amplitude reflects a more tightly bounded range of adaptive capacity, offering enhanced interpretive precision.

High data density within a confined interval signifies that a substantial proportion of the sub-fractal values are concentrated near the central tendency, thereby suggesting a stable and consistent characterization of the system's adaptability. This characteristic is fundamental for calibrating the ACR in a manner that ensures both statistical rigor and predictive reliability. A well-optimized ACR, grounded in high-density intervals, provides a resilient basis for assessing long-term urban adaptability under varying conditions (Weisstein, n.d.; Montgomery & Runger, 2014).

By focusing on regions of maximal data concentration, the methodology effectively filters out outliers and anomalous values, ensuring that the resulting metric reflects a dependable and coherent measure of adaptive behaviour. ***Thus, the ACR is defined as the narrowest range that encompasses the highest data density while satisfying all required statistical constraints. This ensures that the ACR functions not only as a bounded interval of change but as a robust and analytically sound index of urban system resilience.***

To further refine this characterization, a Gaussian (normal) distribution is applied to model the data, allowing for the identification of the central cluster and enhancing the accuracy of range determination.

The probability density function (PDF) of a normal distribution is expressed as:

$$P(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

where x is the variable of interest, μ denotes the mean, and is the standard deviation. The interval exhibiting the highest data density is selected to define the Adaptive Capacity Range (ACR), as it contains the most concentrated portion of the distribution. High data density within this range indicates that the majority of observations are tightly clustered around the mean, signifying a consistent and representative measure of adaptive capacity. This central clustering provides the statistical foundation for the ACR.

Mathematically, this density concentration can be represented by integrating the PDF over the interval surrounding the mean:

$$\int_{\mu-k\sigma}^{\mu+k\sigma} f(x)d(x)$$

This integral quantifies the total probability (or cumulative density) of data points lying within the range, where k is a coefficient determining the breadth of the interval. The selected interval is the one that yields the highest probability mass while maintaining analytical precision, ensuring that the ACR reflects the most stable and recurrent patterns in adaptive behaviour.

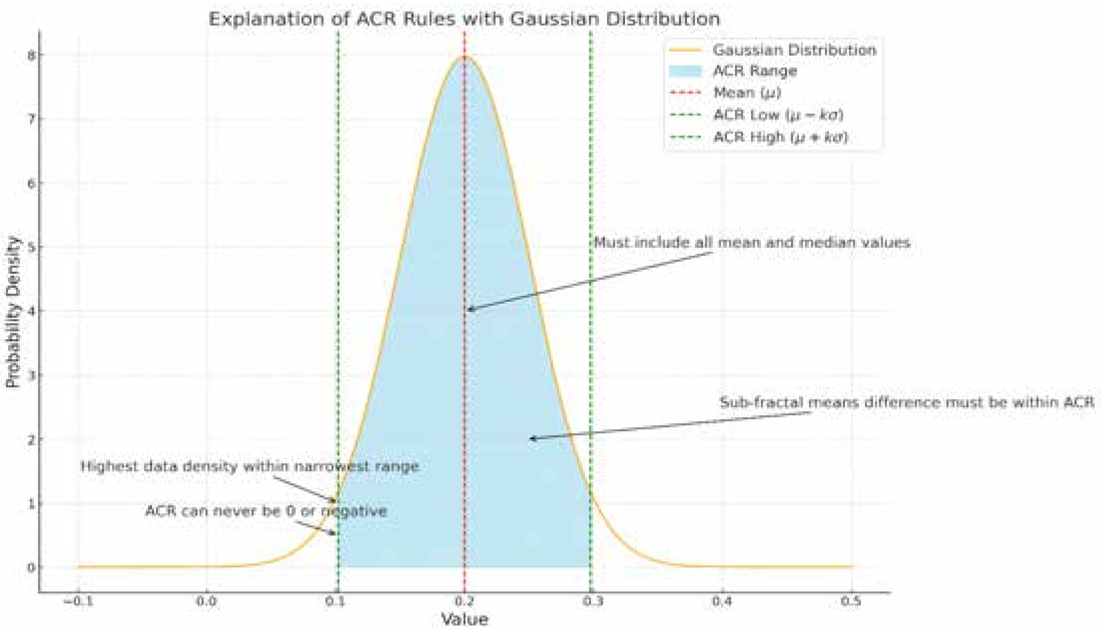

Figure 1. Conceptual Representation of Adaptive Capacity Range (ACR) Conditions Modelled with a Gaussian Distribution.

The graph presented in Figure 1 visualizes four fundamental criteria governing the definition of the Adaptive Capacity Range (ACR) within a Gaussian distribution framework:

• The ACR must remain strictly positive to signify the presence of adaptive potential, as indicated by the distribu-

tion's non-zero probability density.
• It must encompass both the mean and median values across time periods, visually marked by the red dashed line denoting the mean.
• Differences in sub-fractal mean values must fall entirely within the ACR bounds, illustrated by the extent of the blue-shaded region.
• The optimal ACR corresponds to the interval exhibiting the highest data density within the narrowest feasible range, also represented by the blue-shaded area, ensuring both statistical robustness and adaptive relevance.

4. *Sub-Fractal Analysis* of the Izmir City System by Related Plan Boundaries
The study operationalized sub-fractal analysis by structuring network data into spatial maps delineated by historical master plan boundaries. A total of sixteen sub-fractal analysis maps were produced, each corresponding to distinct phases in İzmir's urban planning history. For each planning interval, both the initial (pre-plan) and post-plan states were precisely defined, allowing for the accurate subtraction of sub-fractal box values to generate differential maps. These maps were constructed by calculating the change in sub-fractal dimension values between the two time points, denoted as t_1-t_0.

• Kemal Ahmet Aru's İzmir Plan (1955):
The initial state corresponds to 1950 (Period I), and the post-plan state to 1970 (Period II). The comparative analysis of urban network characteristics under the influence of the 1955 Kemal Aru Plan reveals significant transformations in İzmir's spatial structure over this 20-year span. Using a consistent analytical framework based on 81 sub-fractal boxes per network map (Figure S4), the fractal dimension increased from 1.49 in 1950 to 1.55 in 1970. This upward shift signifies an intensification in spatial complexity, likely driven by increased urban development and diversification. In parallel, the average sub-fractal box value—a finer metric for spatial distribution—rose from 1.188 to 1.341, yielding a net change of 0.152. This trend underscores the densification and increasing emergence of self-similar patterns in the city's infrastructural layout.

• İzmir Metropolitan Municipality (IMM) Plan (1973–1978):
The initial condition is set in 1970 (Period II), with the post-plan state established in 2000 (Period III). The comparative analysis across these three decades under the IMM Plan framework demonstrates continued evolution in urban spatial complexity. The sub-fractal assessment, employing 309 boxes per network map (Figure S5), identified an increase in fractal dimension from 1.43 in 1970 to 1.47 in 2000. Although the growth appears modest, it indicates a steady progression toward greater morphological intricacy. Concurrently, the average box values rose from 1.189 to 1.361, with a differential of 0.172. This increase reflects ongoing densification and the reinforcement of fractal-like, self-similar urban forms, likely shaped by both strategic planning interventions and organic spatial expansion.

• Metropolitan Municipality İzmir Plan (1989):
The initial state for this analysis is defined as 1970 (Period II), while the post-plan condition corresponds to 2000 (Period II). The investigation into urban network transformations under the 1989 İzmir Metropolitan Municipality (IMM) Plan reveals pronounced developments in spatial complexity and structural organization over the 30-year span. Utilizing a detailed grid of 413 sub-fractal boxes per network map (Figure S6), the analysis documented an increase in fractal dimension from 1.51 in 1970 to 1.57 in 2000. This growth reflects an ongoing intensification in the intricacy of the city's spatial configuration, indicating that the urban fabric has evolved in alignment with the transformative objectives embedded in the IMM 1989 planning framework.

In addition to fractal dimension changes, average sub-fractal box values—serving as a granular indicator of spatial distribution and internal variation—also exhibited a marked rise. The average value increased from 1.141 in

1970 to 1.383 in 2000, amounting to a differential of 0.242. This substantial increment underscores the processes of densification and the consolidation of self-similar spatial patterns within İzmir's urban infrastructure. The results highlight the effectiveness of the planning period in fostering more complex and resilient urban morphology.

• ***Metropolitan Municipality İzmir Plan (2012):***
The initial state for this phase of analysis is defined as the year 2000 (Period III), with the post-plan state corresponding to 2020 (Period IV). The evaluation of spatial complexity under the İzmir Metropolitan Municipality (IMM) 2012 Plan reveals continued yet moderated transformation in the city's urban morphology. Employing a high-resolution grid composed of 1,252 sub-fractal boxes per network map (Figure S7), this analysis facilitated a fine-grained assessment of structural evolution over the two-decade span. In 2000, the fractal dimension of the urban network was recorded at 1.39, representing the baseline complexity of the city's spatial structure at the beginning of the planning period. By 2020, this value had modestly increased to 1.41, suggesting a gradual enhancement in urban intricacy. Compared to earlier planning periods, this shift reflects a more restrained trajectory of spatial development—potentially indicative of more regulated urban growth or maturing morphological patterns. Further evidence of densification and internal complexity is provided by the average sub-fractal box values. These values rose from 1.150 in 2000 to 1.316 in 2020, yielding a net difference of 0.166. This increase highlights the persistence of self-similar patterning and the consolidation of spatial structure, albeit progressing at a more tempered rate than in previous decades. Overall, the findings suggest a phase of steady, controlled urban evolution under the guidance of the IMM 2012 Plan.

5. *Sub-Fractal Difference Analysis* of the İzmir Urban System by Planning Periods
The sub-fractal difference analysis delineates the temporal evolution of urban complexity in İzmir across four major planning intervals: the 1955 Kemal Aru Plan, the 1973–78 İzmir Metropolitan Municipality (IMM) Plan, the 1989 IMM Plan, and the 2012 IMM Plan. This assessment is based on variations in both the mean and median values of sub-fractal differences, which are visualized in Figure S8.

The mean values of sub-fractal differences, indicated by the solid blue line, exhibit the following sequence: 0.15 for the 1955 plan, 0.17 for the 1973–78 plan, 0.24 for the 1989 plan, and 0.17 for the 2012 plan. Correspondingly, the median values—depicted by the red dashed line—are recorded as 0.15 (1955), 0.09 (1973–78), 0.11 (1989), and 0.13 (2012). All measured values for both mean and median fall within the designated Adaptive Capacity Range (ACR) of 0 to 0.25. This alignment underscores a consistent pattern of manageable complexity shifts, indicating that despite varying degrees of planning intervention and urban growth, the city's adaptive transformation has remained within statistically and structurally interpretable bounds.

• ***Kemal Aru İzmir Plan (1955):***
Sub-fractal difference analysis conducted within the planning boundary of the Kemal Aru 1955 Plan, comparing network configurations from 1950 and 1970, revealed that 37 out of 81 analysed grid boxes fell within the Adaptive Capacity Range (ACR) of 0 to 0.25. This finding suggests substantial adaptive potential embedded within the urban system during this period (Figure S9). The frequency distribution of sub-fractal differences was assessed using a bin width of 0.05. The proportion of values within the ACR range accounted for 52.44% of the total dataset, reflecting a high degree of adaptability. A Gaussian curve was fitted to the data, yielding a mean (μ) of 0.15 and a standard deviation of 0.45. The distribution's peak corresponded precisely with the mean value, confirming central clustering. Both mean and median values were well-contained within the defined ACR, affirming its statistical robustness. The regression model exhibited an exceptionally high degree of fit, with an R^2 value of 0.9998, indicating an almost perfect linear relationship between the bin means and the fitted curve. Additionally, the

Q-Q plot for this period demonstrated close alignment between empirical quantiles and the theoretical normal distribution, further validating the Gaussian fit (Figure S10).

• ***İzmir Metropolitan Municipality Plan (1973–1978):***
An analogous sub-fractal difference analysis was performed within the boundaries of the IMM 1973–78 Plan, comparing urban network data from 1970 and 2000. Of the 309 sub-fractal box values examined, 127 were found to fall within the ACR (0–0.25), indicating that a considerable segment of the urban landscape demonstrated adaptive spatial characteristics (Figure S11). The distribution analysis revealed that 44.52% of all values resided within the ACR, slightly lower than the 1955 period but still reflective of a resilient urban system. The Gaussian distribution fitted to the data produced a mean (μ) of 0.17 and a standard deviation (σ) of 0.44, with the distribution peaking at 0.20. This central tendency illustrates a notable concentration of difference values within the adaptive threshold. The regression fit demonstrated strong linearity, with an R^2 value of 0.9998, validating the model's precision. Additionally, the Q-Q plot confirmed that empirical and theoretical quantiles were closely aligned, supporting the statistical consistency of the Gaussian approximation (Figure S12).

• ***İzmir Metropolitan Municipality Plan (1989):***
Sub-fractal difference analysis conducted within the spatial scope of the IMM 1989 Plan aimed to assess the impact of planning decisions on the city's spatial complexity. A total of 413 sub-fractal box values were analysed, of which 166 fell within the Adaptive Capacity Range (ACR) of 0 to 0.25—marking a substantial expression of the city's capacity to absorb change and maintain structural functionality in the face of planned interventions or emerging urban pressures (Figure S13). The proportion of values within the ACR constituted 42.03% of the dataset, indicating moderate but meaningful adaptability. A Gaussian distribution was fitted to the difference data, yielding a mean (μ) of 0.24 and a standard deviation of 0.44, with the distribution's peak occurring at 0.26. These statistical parameters suggest that the bulk of sub-fractal variation remained concentrated around the ACR's upper threshold, highlighting the persistence of adaptability within an increasingly complex urban form. The model fit was highly reliable, confirmed by an R^2 value of 0.9998. Moreover, the Q-Q plot demonstrated a strong correspondence between empirical and theoretical quantiles, reinforcing the validity of the normal distribution fit (Figure S14).

• ***İzmir Metropolitan Municipality Plan (2012):***
The city-wide sub-fractal analysis for the IMM 2012 Plan evaluated spatial complexity changes using 1,252 box values, providing a comprehensive account of adaptive patterns over time. Of these, 603 boxes displayed sub-fractal differences within the defined ACR (0–0.25), signifying a widespread capacity for structural adaptation and resilience across İzmir's urban landscape (Figure S15). The results indicated that 52.07% of the observed values were contained within the ACR, reflecting a high degree of adaptability during this planning phase. The application of a Gaussian fit produced a mean (μ) of 0.17 and a standard deviation of 0.45, with the peak of the distribution observed at 0.15. These outcomes suggest a well-centred and consistent distribution of difference values within the ACR, reinforcing the notion of sustained adaptability in the city's urban morphology. As with previous periods, the model demonstrated exceptional precision, with an R^2 value of 0.9998, and the Q-Q plot confirmed the close alignment between observed and theoretical quantiles, validating the Gaussian approximation (Figure S16).

- **Regional Sub-Fractal Analysis within the 2012 IMM Planning Boundary**
The 2012 İzmir Metropolitan Municipality Plan was subjected to a detailed regional analysis to evaluate spatial adaptability using sub-fractal difference values. This assessment involved calculating the mean, median, and standard deviation for each region and determining the proportion of data falling within the defined Adaptive Capacity Range (ACR) of 0 to 0.25 (Figure S17).

o ***Northern İzmir:*** Of the 228 grid boxes analysed, 73 (37.99%) fell within the ACR. The sub-fractal difference values exhibited a mean of 0.17, a median of 0.10, and a standard deviation of 0.62. The Gaussian model fit was confirmed with a robust R^2 value of 0.9998, indicating a strong correlation (Figure S18).

o ***Southern İzmir:*** In this region, 48 out of 131 boxes (40.15%) were within the ACR. The mean difference was 0.20, the median was 0.16, and the standard deviation measured at 0.46. The model's accuracy was validated by an R^2 value of 0.9998 (Figure S19).

o ***Western İzmir:*** Here, 82 of 191 boxes (46.88%) were within the ACR. The statistical outputs showed a mean of 0.19, a median of 0.15, and a standard deviation of 0.39. The Gaussian model demonstrated a strong fit with an R^2 of 0.9998 (Figure S20).

o ***Eastern İzmir:*** Of the 148 boxes assessed, 66 (51.01%) fell within the ACR. The region recorded a mean of 0.20, a median of 0.17, and a standard deviation of 0.42. The fit quality was high, supported by an R^2 value of 0.9998 (Figure S21).

o ***Tourism Region (Gümüldür Area):*** In this area, 17 out of 48 boxes (42.86%) were within the ACR. The mean sub-fractal difference was 0.14, with a median of 0.12 and a standard deviation of 0.46. The Gaussian fit held with an R^2 of 0.9998, confirming model validity (Figure S22).

o ***Central İzmir:*** Central İzmir demonstrated the strongest adaptive capacity among all regions, with 313 of 506 boxes (64.69%) falling within the ACR. The mean difference was 0.14, the median was 0.13, and the standard deviation was 0.37. The correlation between observed and theoretical values remained strong, with an R^2 of 0.9998 (Figure S23).

Collectively, the regional analysis confirmed the effectiveness of the 0–0.25 Adaptive Capacity Range in capturing the city's multifaceted ability to accommodate urban transformation. The consistent Gaussian distributions, relatively moderate standard deviations, and uniformly high R^2 values across all sub-regions reinforce the reliability of the model in representing İzmir's systemic resilience to planning and environmental pressures.

6. Discussion on the *Adaptive Capacity Range* of İzmir as a *Complex Physical System*

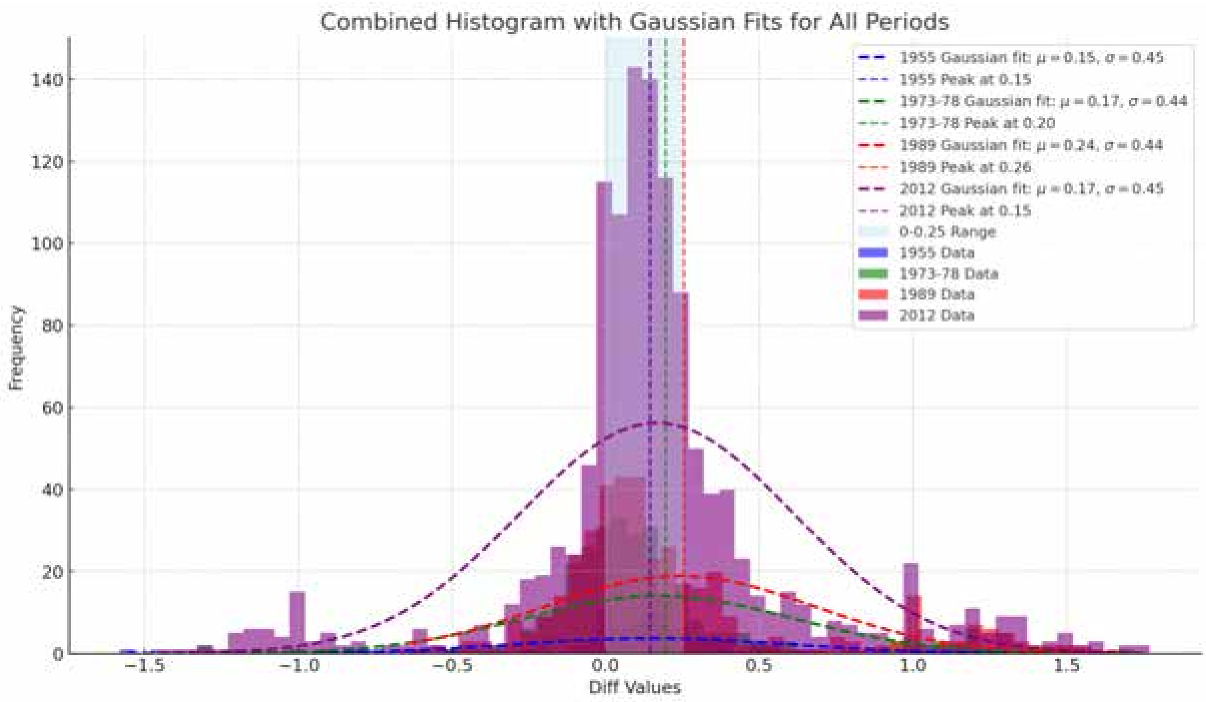

Figure 2. Combined Histogram with Gaussian Fits for periods (Distribution of Metric Differences across All Time Periods and Regions)

The longitudinal analysis of İzmir's urban development, employing sub-fractal and difference analysis methodologies applied to its road network, offers a nuanced understanding of the city's Adaptive Capacity Range (ACR) and its implications for resilient urban planning. This investigation spans network configurations from 1950 to 2020, with fractal dimension values extracted to trace the evolving complexity of İzmir's spatial structure in response to successive planning initiatives. Across all periods, the ACR was consistently situated within the interval of 0 to 0.25, suggesting a moderate but structurally meaningful degree of adaptability. This bounded range underscores the city's ability to reorganize and sustain infrastructural functionality amid socio-spatial transformations (see Figure 1).

Methodologically, the study synthesizes sub-fractal computation with difference analysis, grounded in the mathematical logic of fractal geometry and the dynamic principles of complex systems theory. Originally formulated by Mandelbrot (1982), fractal analysis offers a theoretical lens for interpreting the self-similar and scale-invariant patterns that characterize urban morphology. This paradigm is reinforced by Batty (2005), who posits that urban spatial forms inherently manifest fractal behavior, enabling their internal structural dynamics to be captured through rigorous quantitative measures.

The conceptualization of adaptive capacity is enriched by interdisciplinary analogies, particularly from statistical mechanics and quantum theory. In statistical mechanics, the notion of phase transitions and critical thresholds demonstrates how marginal perturbations can catalyze disproportionate systemic reconfigurations—a dynamic mirrored in urban environments, where minor policy adjustments or spatial interventions can induce marked transformations in form and function (Batty, 2005). Similarly, the probabilistic ontology of quantum systems provides an apt metaphor for urban complexity, wherein outcomes are shaped not by deterministic rules but by fluctuating probabilities and interdependent interactions (Mandelbrot, 1982).

Through this interdisciplinary integration, the research frames cities as complex adaptive systems and positions the ACR as a predictive metric that operationalizes resilience. Functioning as a quantitative analogue to critical thresholds in physical systems, the ACR encapsulates the city's adaptive behavior by incorporating both mean and median sub-fractal difference values. This dual inclusion ensures that the analysis reflects central tendencies while preserving sensitivity to distributional asymmetries, thereby offering a comprehensive representation of urban adaptability. Such a framework aligns with broader ecological and socio-technical theories of resilience, notably those advanced by Folke et al. (2002), which emphasize the importance of systemic flexibility and transformative potential in the face of uncertainty.

The regional sub-fractal analysis further illuminates the spatial heterogeneity of İzmir's adaptive capacity, revealing distinct variability in resilience across different urban zones. Northern and Southern İzmir exhibit moderate adaptability, with 37.99% and 40.15% of data points respectively falling within the Adaptive Capacity Range (ACR). The Western and Eastern districts display comparatively higher resilience, with ACR coverage rates of 46.88% and 51.01%. Notably, Central İzmir demonstrates the most robust adaptive performance, with 64.69% of its data residing within the ACR. This pattern of variation substantiates the concept of spatial heterogeneity in urban resilience, as articulated by Folke et al. (2002), which posits that adaptive capacity is unevenly distributed across the urban fabric, shaped by historical, socio-economic, and infrastructural asymmetries.

The practical implications of these findings extend beyond theoretical analysis, offering actionable tools for urban planners and policy designers. By systematically quantifying urban complexity and delineating adaptive thresholds, this methodology enables planners to anticipate structural vulnerabilities and design targeted interventions. Such an approach aligns with broader urban resilience discourses, emphasizing the pivotal role of adaptive planning in guiding sustainable urban transformation (Batty, 2005). Moreover, the methodological integration of sub-fractal analysis with principles derived from theoretical physics highlights a novel and interdisciplinary pathway for urban systems modeling—bridging statistical mechanics, complexity theory, and spatial analytics to forecast urban evolution.

In conclusion, this study reinforces the necessity of embedding complex systems theory into the praxis of urban planning to effectively contend with the nonlinear and unpredictable nature of urban growth. The results suggest that urban environments characterized by higher degrees of spatial complexity, as measured through detailed fractal metrics, exhibit enhanced adaptive capacities—positioning them to more effectively absorb and respond to future socio-environmental challenges. Looking forward, the extension of this analytical framework to other metropolitan contexts, coupled with the integration of real-time geospatial and socio-economic data, holds significant potential for advancing our understanding of urban adaptability and fostering the development of more resilient and sustainable urban systems.

7. Conclusion

The analytical approach employed in this study—integrating sub-fractal and difference analysis—draws theoretical grounding from Mandelbrot's (1982) seminal contributions to fractal geometry and Goldenfeld's (1992) investigation of phase transitions within statistical mechanics. By transposing the conceptual apparatus of statistical physics to the domain of urban studies, this research establishes a robust analytical framework for interpreting and managing the nonlinear dynamics of urban expansion and adaptive transformation. The methodologies illustrate how micro-scale fluctuations—such as minor infrastructural or morphological alterations—can precipitate pronounced changes at the macro-urban scale, reflecting the emergent behavior described by Yeomans (1992) in relation to the aggregation of microscopic interactions into large-scale phenomena.

The quantification of Adaptive Capacity Range (ACR) within this context exemplifies a productive intersection between theoretical physics and spatial urban analytics. This convergence equips urban planners with advanced tools for anticipating and steering urban morphological evolution. Consistent with the principles of complex systems theory—which conceptualizes cities as adaptive, self-organizing entities (Bar-Yam, 1997)—this research applies fractal diagnostics to capture the intrinsic irregularities and hierarchies of urban form. In doing so, it contributes empirical support to Batty's (2005) assertion that cities inherently exhibit fractal structures, and that these patterns can be rigorously interrogated to illuminate underlying processes of spatial complexity and systemic adaptability.

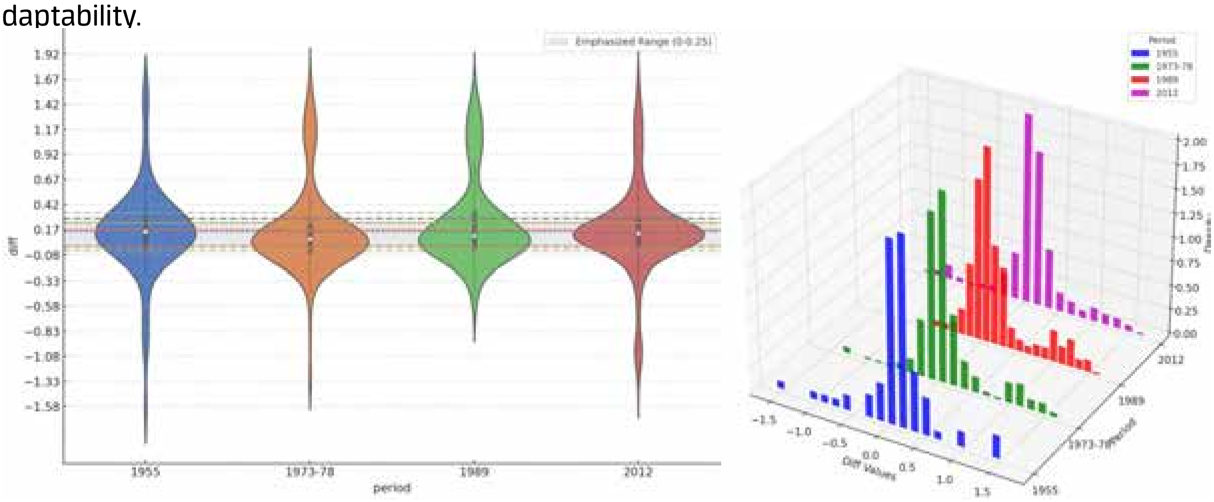

Figure 3. Density Distributions of Sub-Fractal Difference Values Across All Periods and Regions

This figure depicts the distributional characteristics of sub-fractal difference values across all examined temporal and spatial domains in İzmir. Utilizing both violin plots and 3D histograms, the visualization reveals the variability, skewness, and concentration of data points. Central to this representation is the highlighted 0–0.25 range, corresponding to the Adaptive Capacity Range (ACR), which encompasses the majority of observed values. This reinforces the notion that urban systems exhibit measurable, intrinsic adaptability, which can be captured through sub-fractal metrics. Beyond visual analysis, the findings underscore the methodological significance of ACR as a robust and transferable metric for urban adaptability. The empirical demonstration that ACR is not only computable but also meaningful within the urban planning context lays the groundwork for methodological standardization. Such a framework could be extrapolated to diverse urban contexts, enabling comparative resilience assessments and fostering evidence-based planning strategies. This aligns with Portugali's (2012) call for integrating complexity science into urban design and policy, advancing a data-driven, systemic approach to fostering urban resilience.

Future Directions

Future research should extend the application of sub-fractal and difference analysis to a wider range of urban contexts to evaluate the universality of the Adaptive Capacity Range (ACR). Comparative studies across cities with differing scales, complexities, and socio-economic profiles will help determine whether İzmir's patterns are generalizable or context-specific (Bettencourt, 2013; Arcaute et al., 2015; Ribeiro et al., 2019). Incorporating real-time data—such as traffic, mobility, economic activity, and environmental conditions—can enhance model accuracy and responsiveness (Calabrese et al., 2013; Hasan et al., 2013). Such integration would enable more dynamic assessments of adaptability, particularly during disruptions (Lu et al., 2016). The rise of smart cities offers further opportunities to leverage IoT technologies and big data analytics for tracking and modeling urban complexity (Batty, 2018). Combined with AI and machine learning, these tools can identify patterns, predict growth, and simulate policy outcomes (Harris, 1991).

Interdisciplinary collaboration is essential for advancing holistic models that incorporate social, economic, and environmental dimensions of resilience (Ahern, 2011; Folke et al., 2010). Engagement with planners, policymakers, and communities will also be key in translating academic findings into practical tools (Portugali, 2012; Adger et al., 2005). In sum, expanding geographic scope, integrating dynamic data, leveraging AI, and fostering cross-sector collaboration will deepen our understanding of urban adaptive capacity and support more resilient and sustainable urban development.

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Appendices

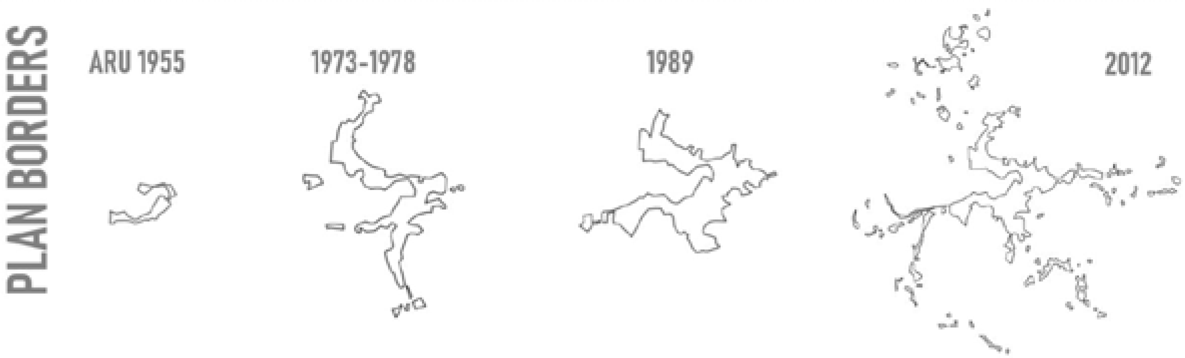


Figure S1. Plan borders for sub-fractal analysis process by periods.

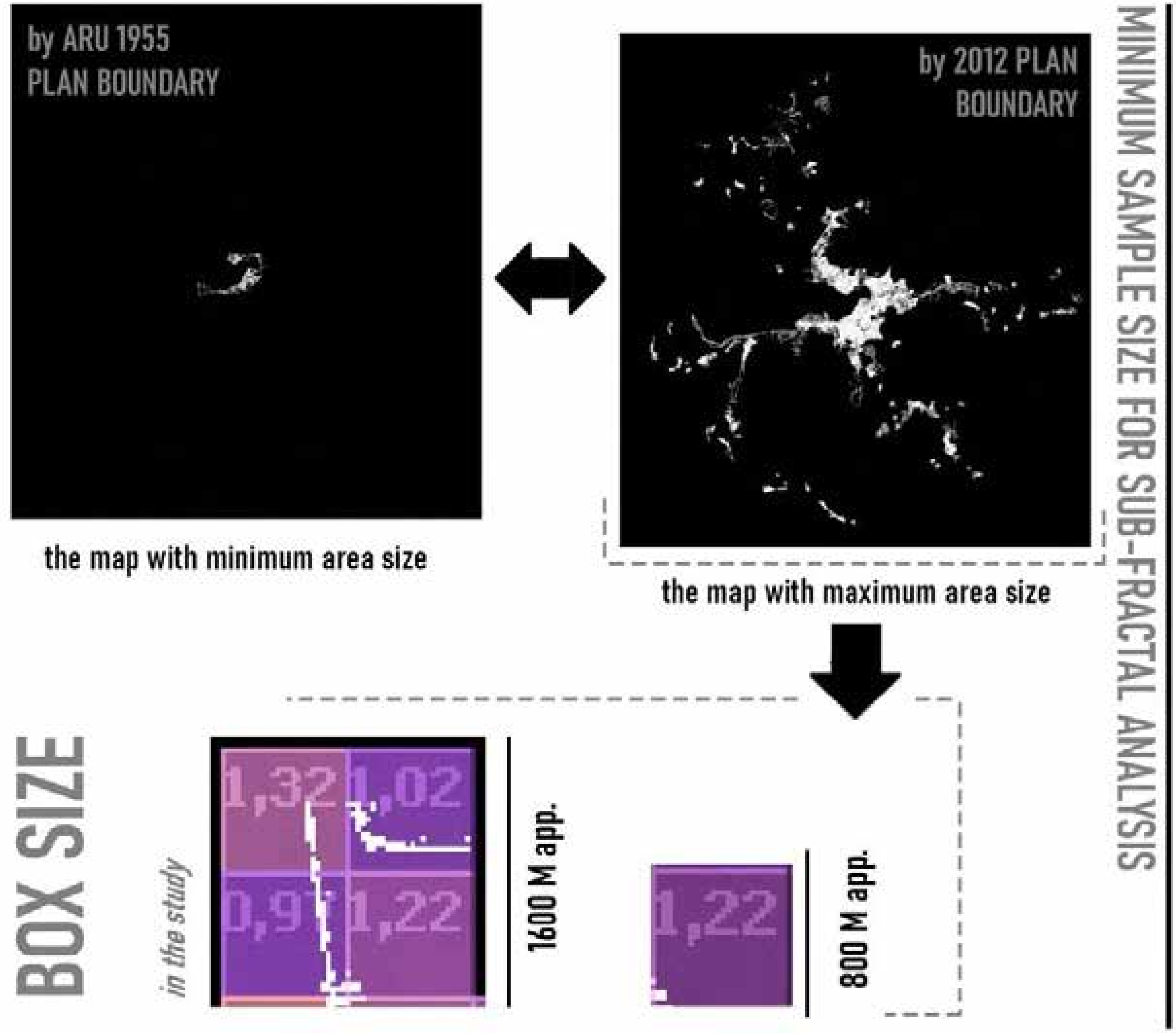


Figure S2. Minimum sample size algorithm for sub-fractal analysis

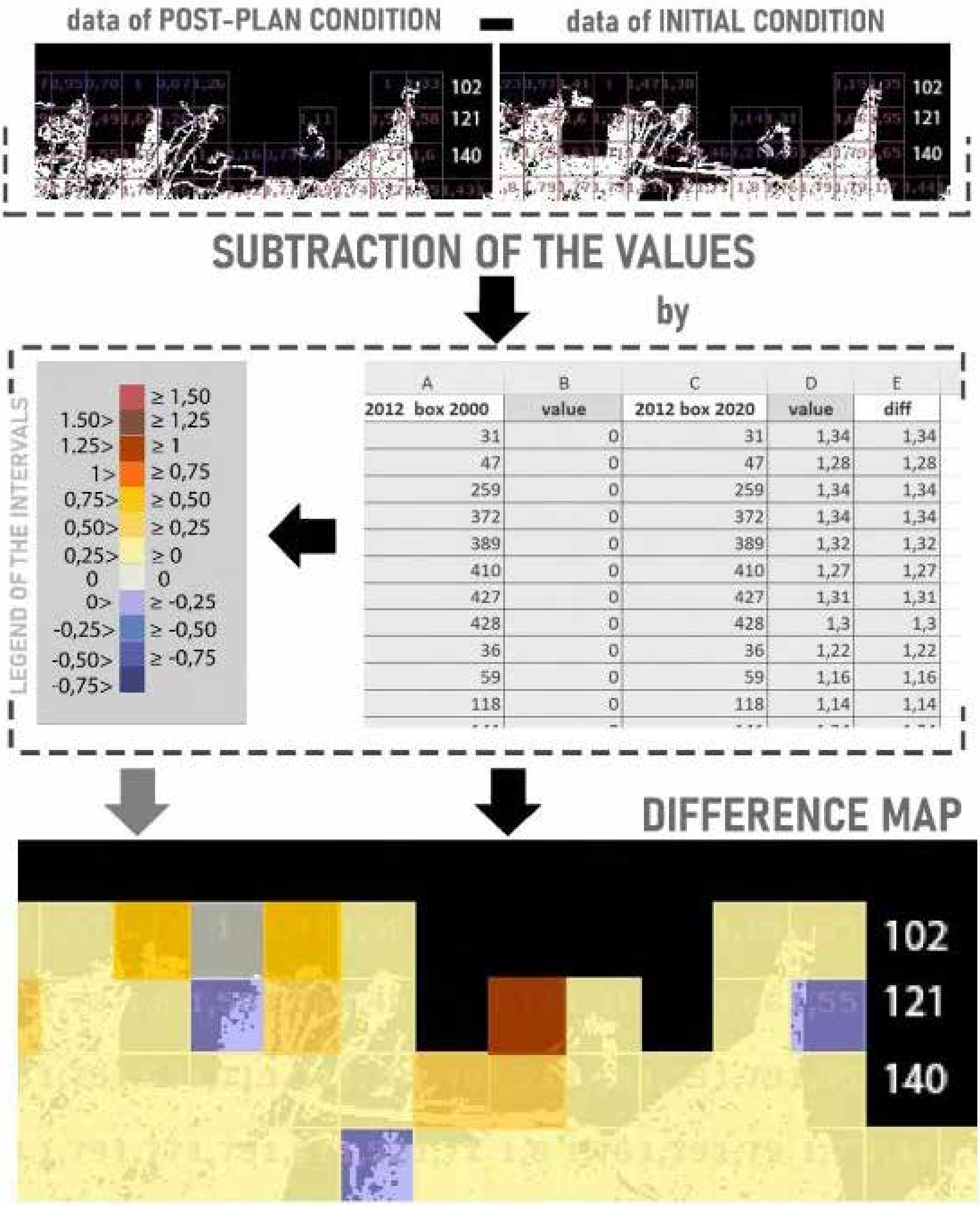


Figure S3. Flow chart showing difference map production.

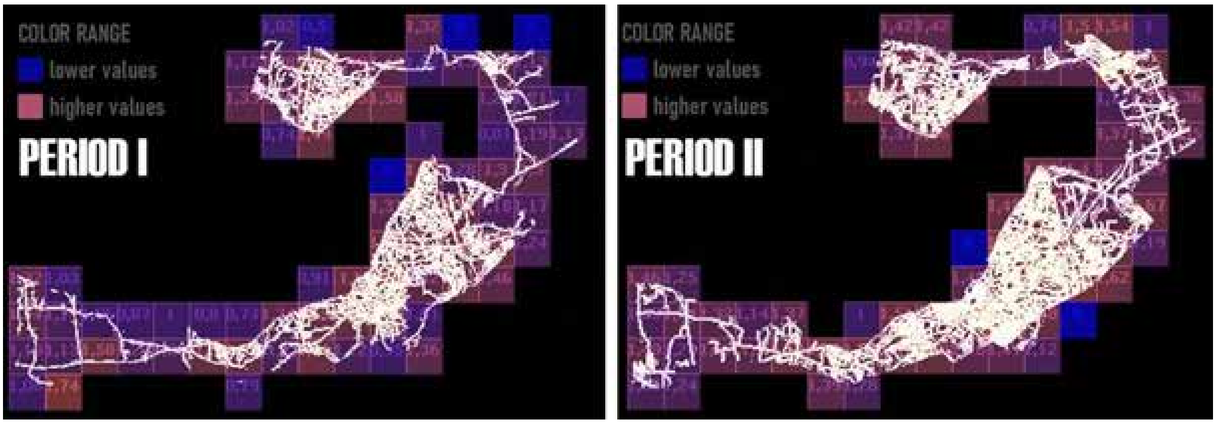


Figure S4. Sub-Fractal analysis of pre-plan state and post plan state of the İzmir plan of Kemal Ahmet

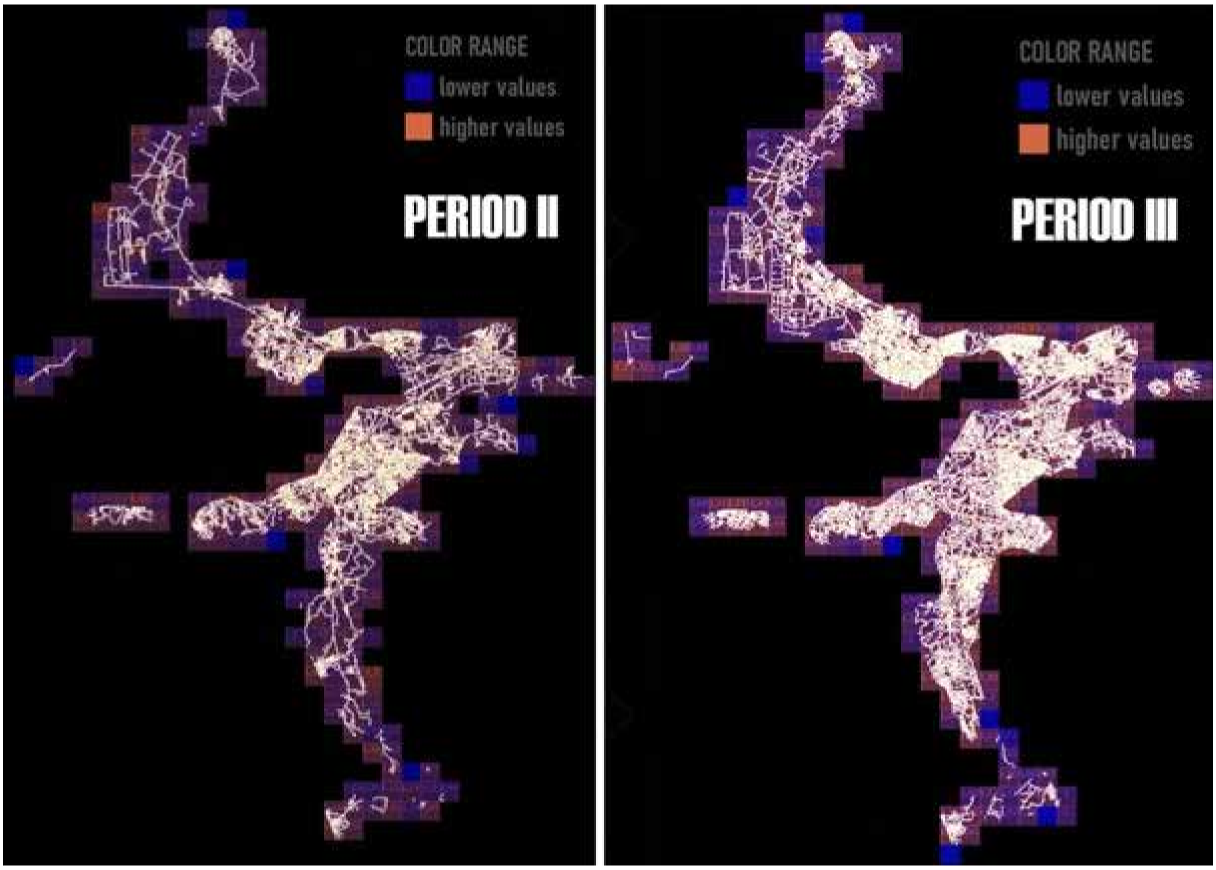
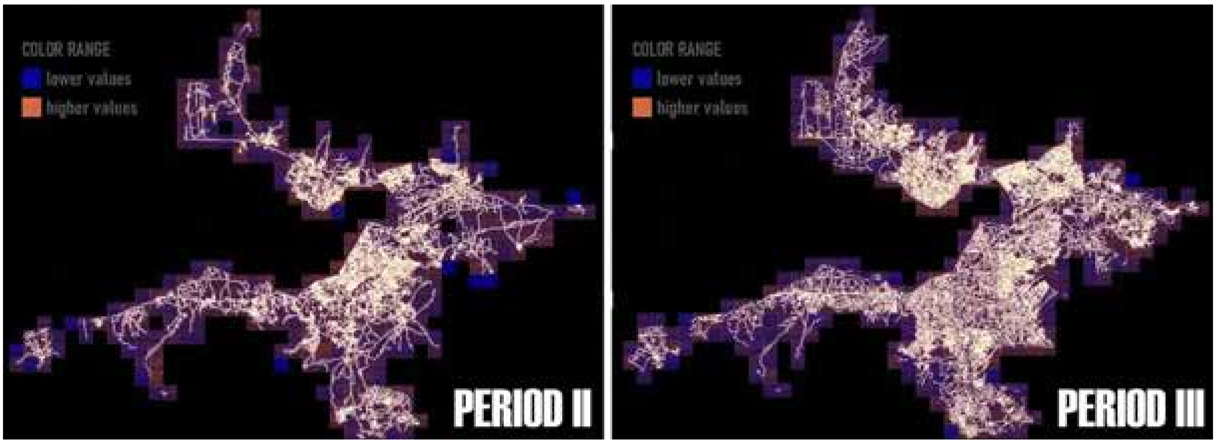


Figure S5. Sub-Fractal analysis of pre-plan state of the İzmir plan of metropolitan municipality, 1973-78



FigureS6. Sub-Fractal analysis of pre-plan state of the İzmir plan of metropolitan municipality, 1989.

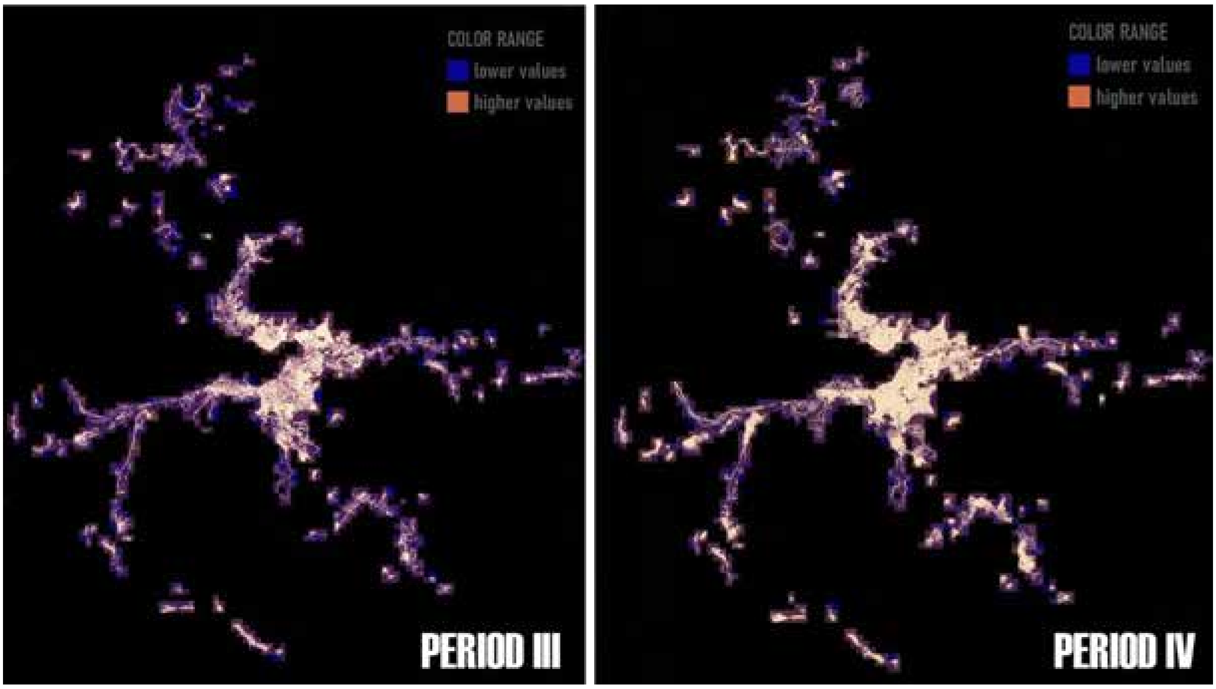


Figure S7. Sub-Fractal analysis of pre-plan state of the İzmir plan of metropolitan municipality, 2012.

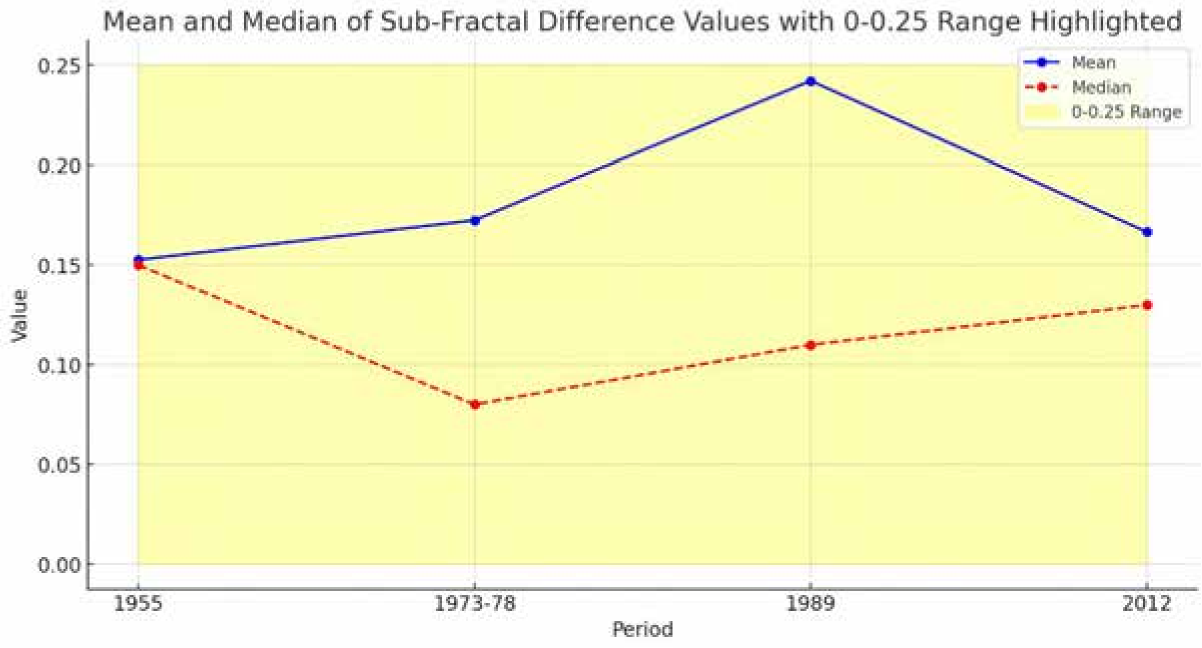


Figure S8. Mean and Median of Sub-Fractal Difference Values with Adaptive Capacity Range

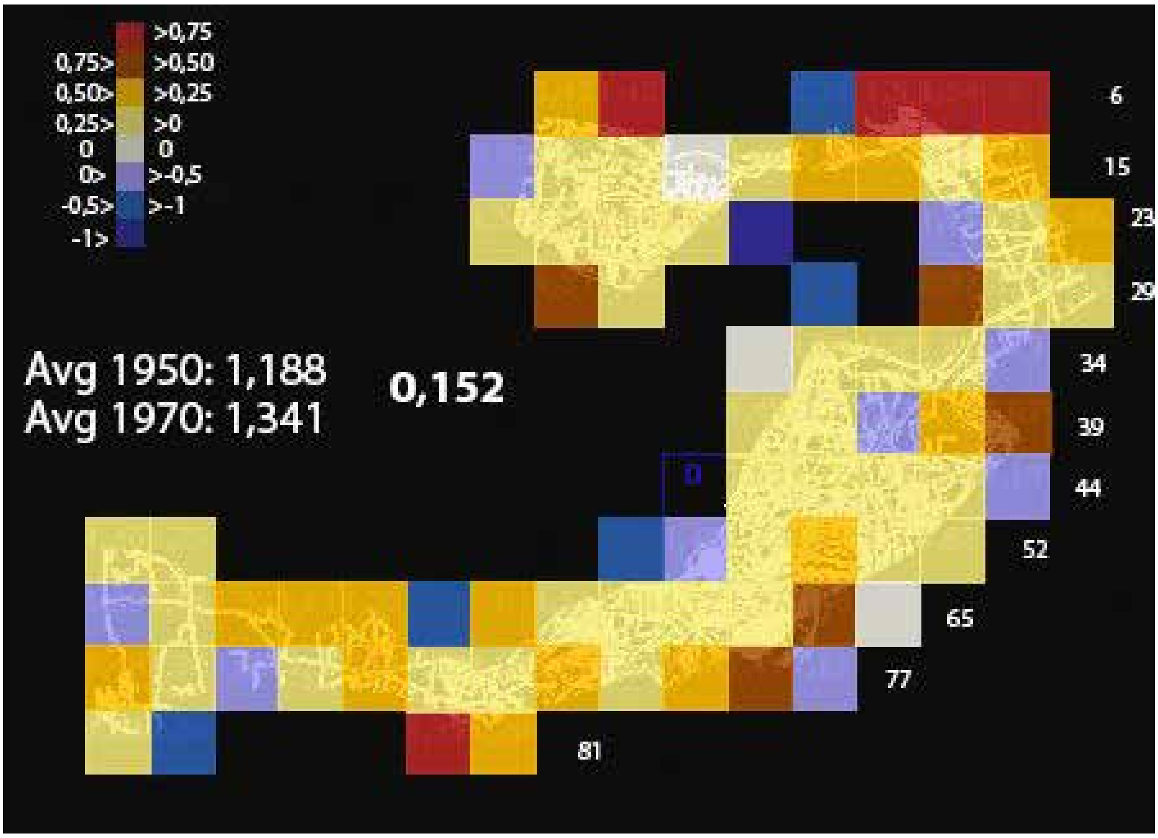


Figure S9. Difference Analysis of Periods I and II by Kemal Ahmet Aru Plan, 1955.

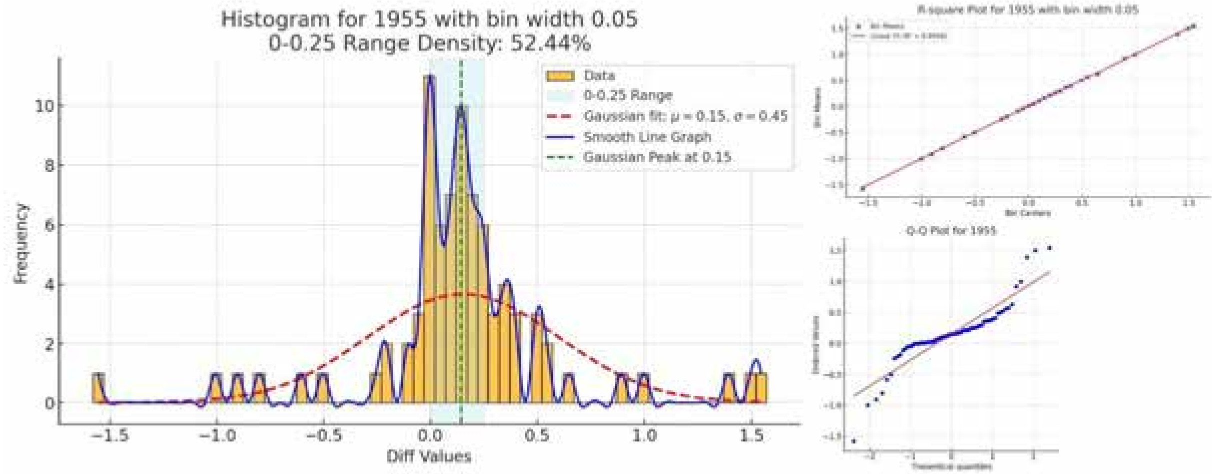


Figure S10. Statistical Analysis of Sub-Fractal Difference Values for the İzmir Plan of Kemal Aru 1955: Histogram with the Range Density, Gaussian Fit, QQ Plot and R^2 value.

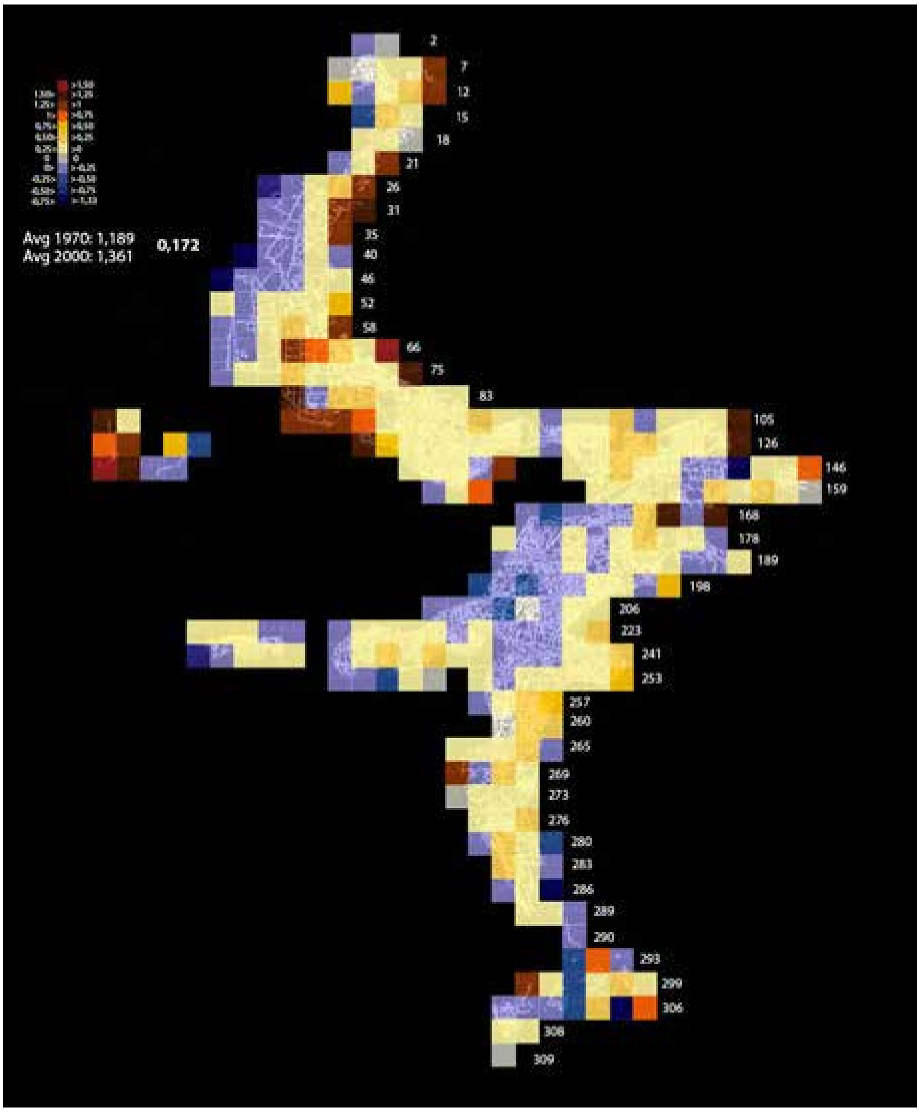


Figure S11. Difference Analysis of Periods II and III by the İzmir plan of metropolitan municipality,1973-78.

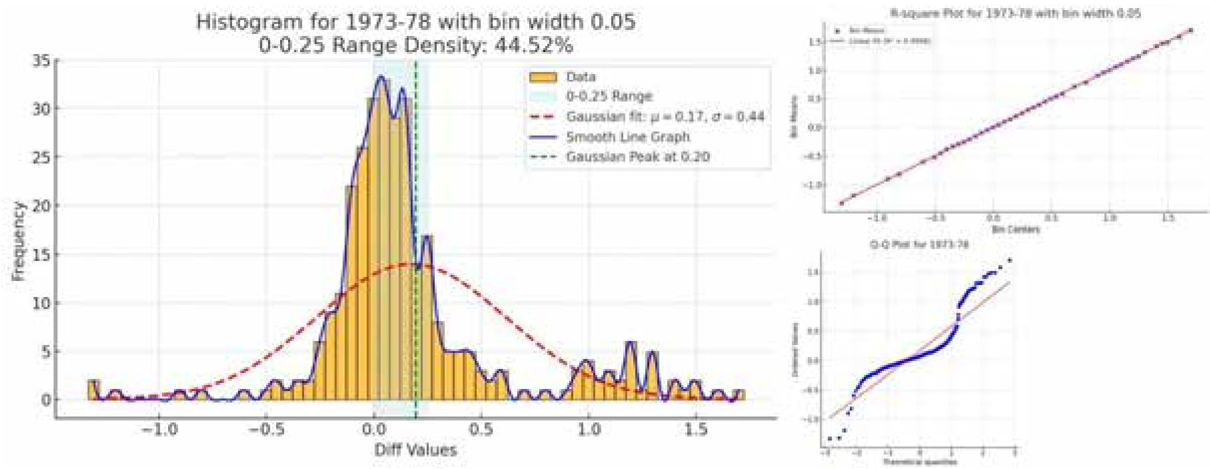


Figure S12. Statistical Analysis of Sub-Fractal Difference Values for the Izmir Plan of Metropolitan Municipality, 1973-78: Histogram with the Range Density, Gaussian Fit, QQ Plot and R^2 value.

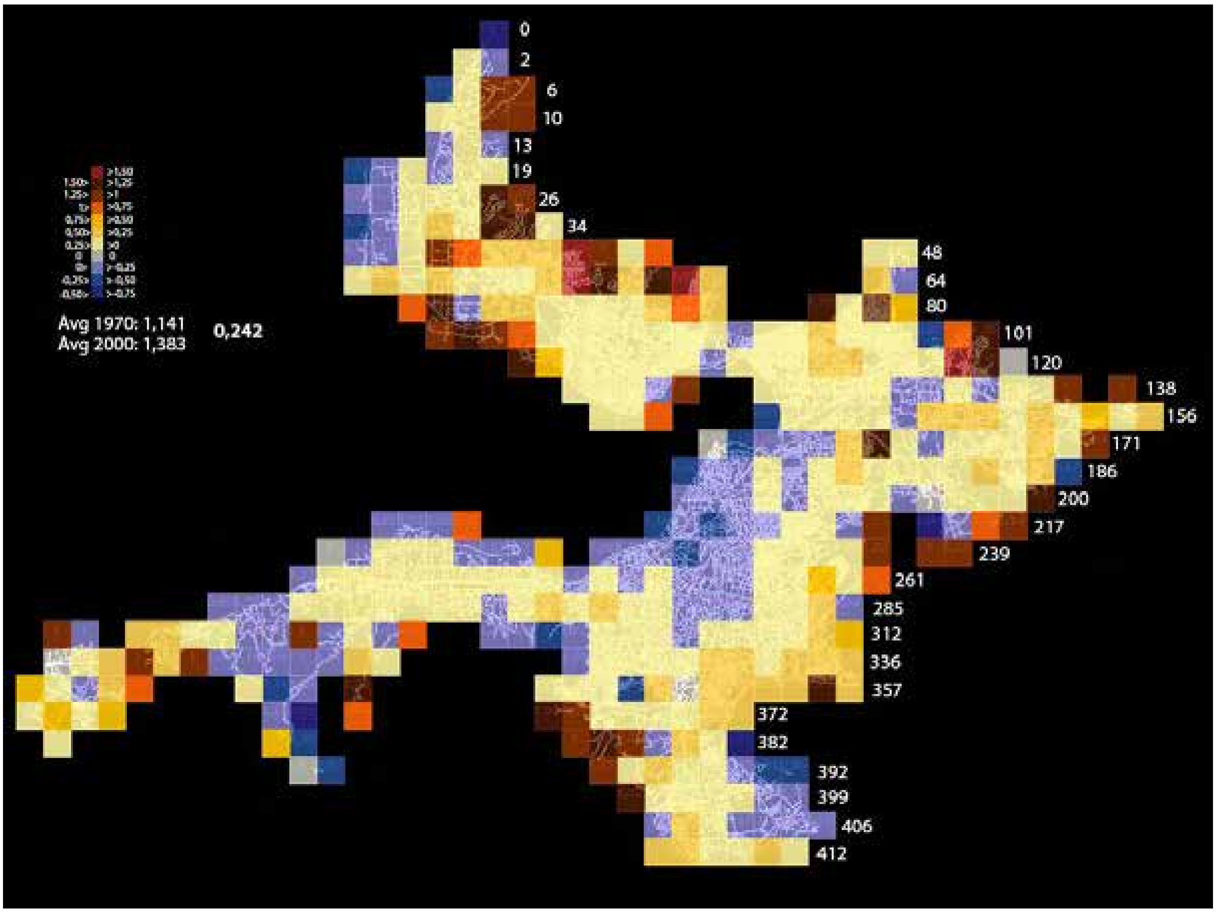


Figure S13. Difference Analysis of Periods II and III by the İzmir plan of metropolitan municipality, 1989.

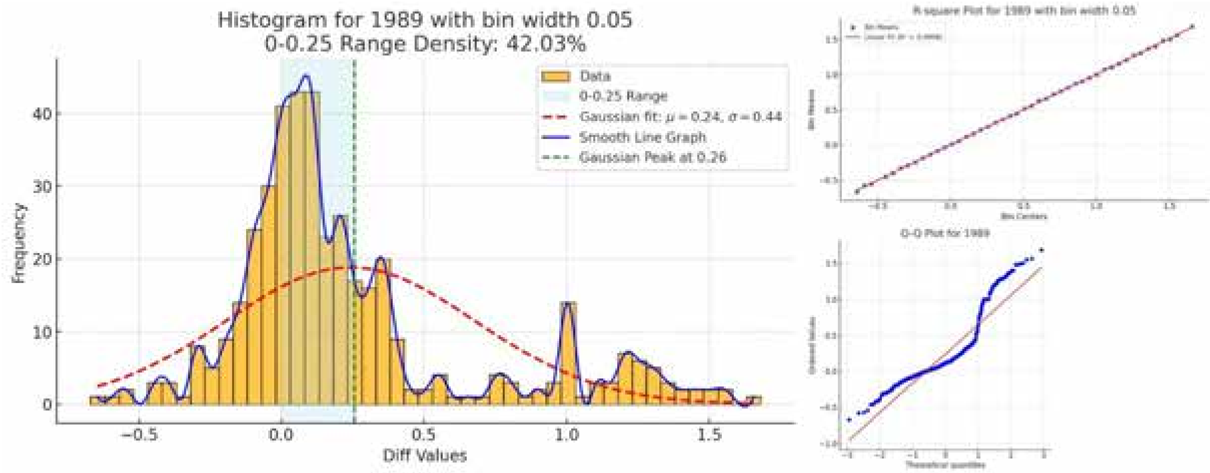


Figure S14. Statistical Analysis of Sub-Fractal Difference Values for the Izmir Plan of Metropolitan Municipality, 1989: Histogram with the Range Density, Gaussian Fit, QQ Plot and R^2 value.

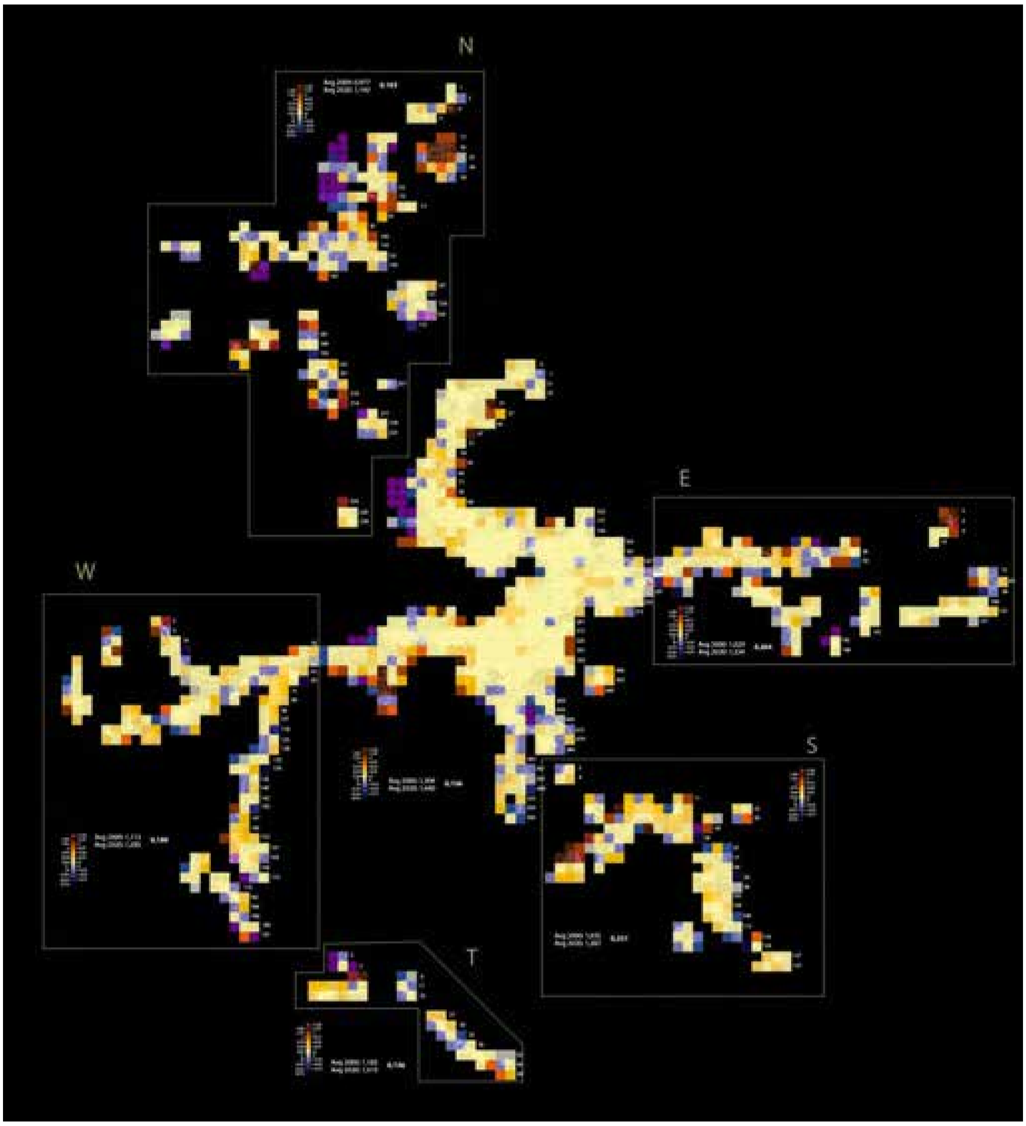


Figure S15. Difference Analysis of Periods III and IV by the İzmir plan of metropolitan municipality,2012.

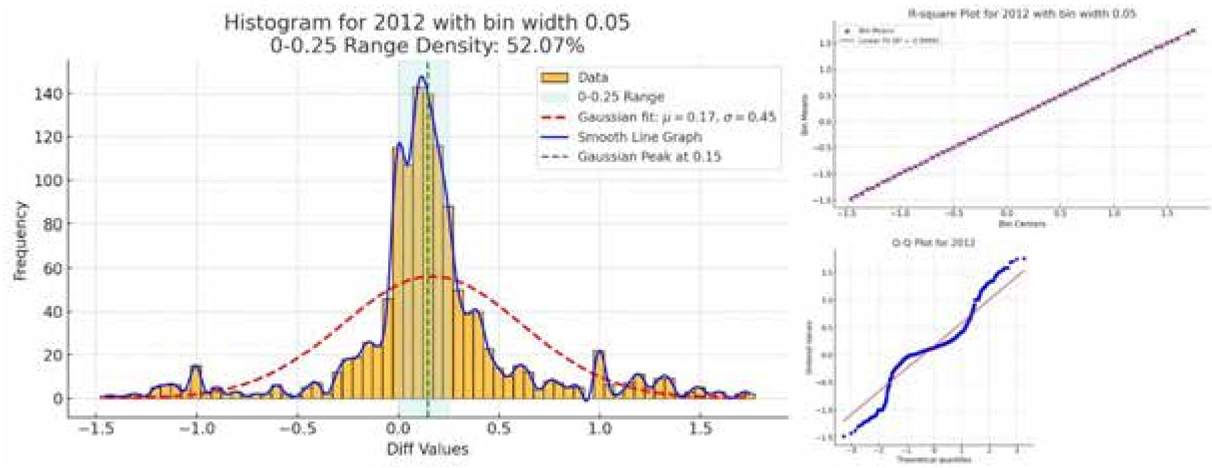


Figure S16. Statistical Analysis of Sub-Fractal Difference Values for the Izmir Plan of Metropolitan Municipality, 2012: Histogram with the Range Density, Gaussian Fit, QQ Plot and R^2 value.

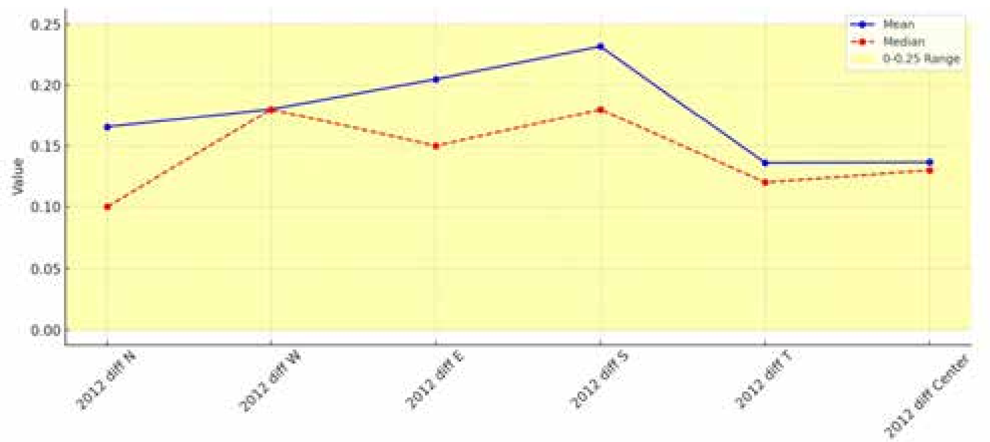


Figure S17. Means and Medians of Sub-Fractal Difference Values of 2012 Plan Period's Regions with Adaptive Capacity Range Highlighted.

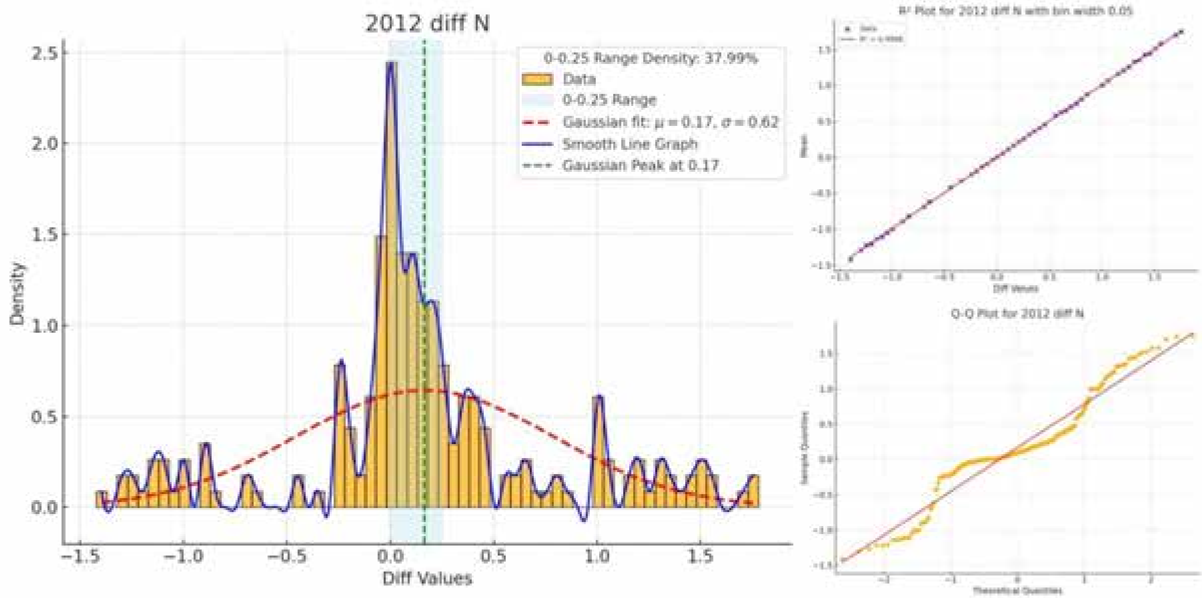


Figure S18. Statistical Analysis of Sub-Fractal Difference Values for the Izmir Plan of Metropolitan Municipality, 2012 (Northern Region): Histogram with the Range Density, Gaussian Fit, QQ Plot and R^2 value.

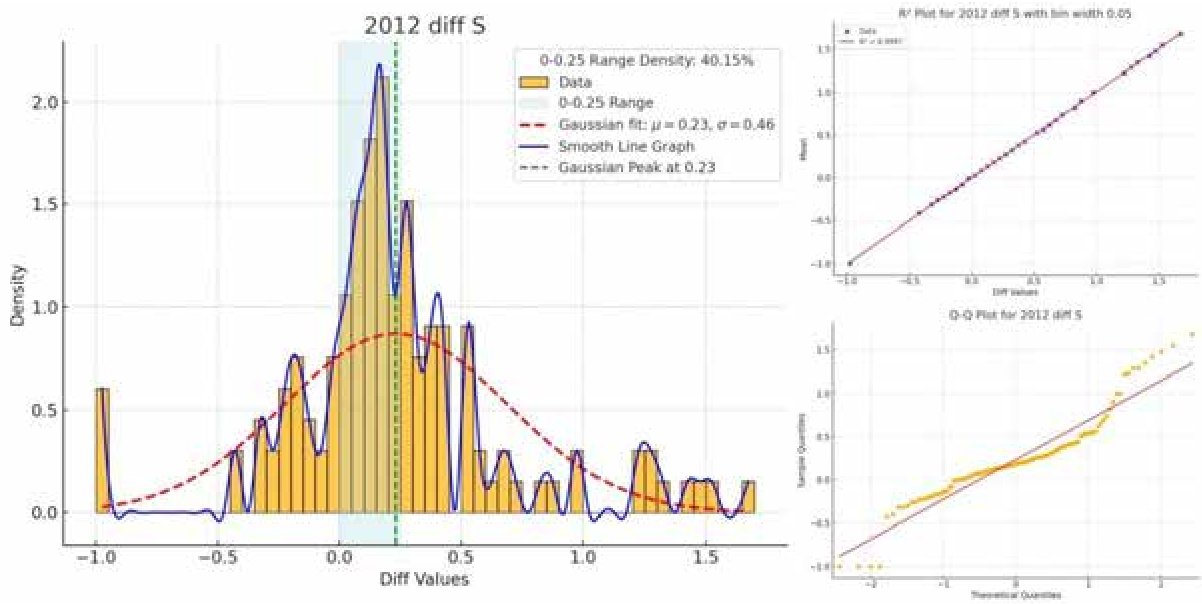


Figure S19. Statistical Analysis of Sub-Fractal Difference Values for the Izmir Plan of Metropolitan Municipality, 2012 (Southern Region): Histogram with the Range Density, Gaussian Fit, QQ Plot and R^2 value.

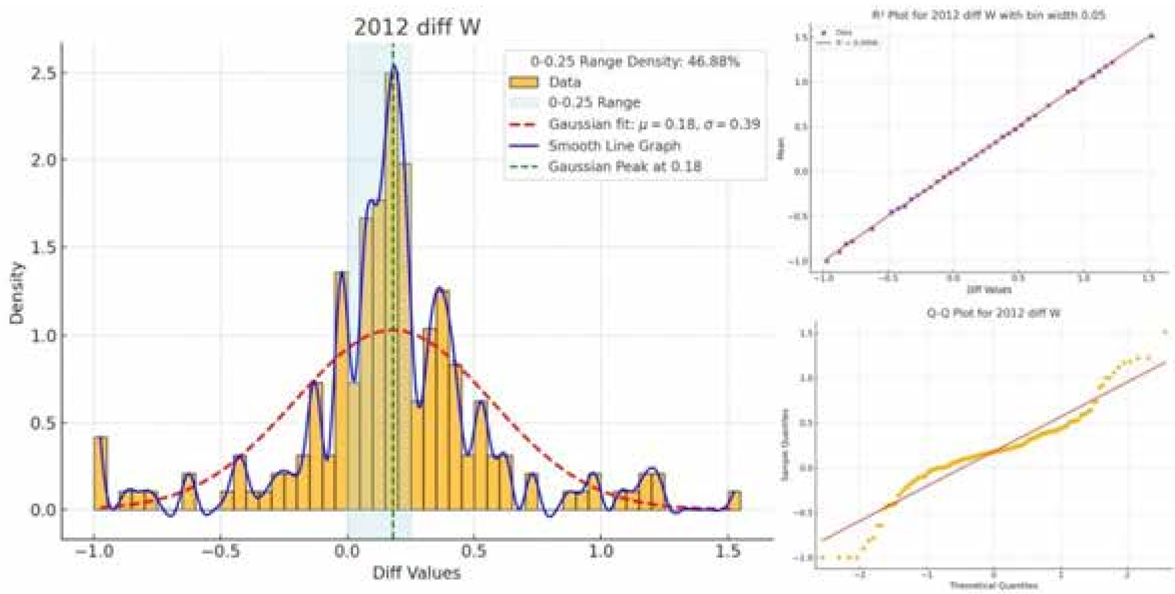


Figure S20. Statistical Analysis of Sub-Fractal Difference Values for the Izmir Plan of Metropolitan Municipality, 2012 (Western Region): Histogram with the Range Density, Gaussian Fit, QQ Plot and R^2 value.

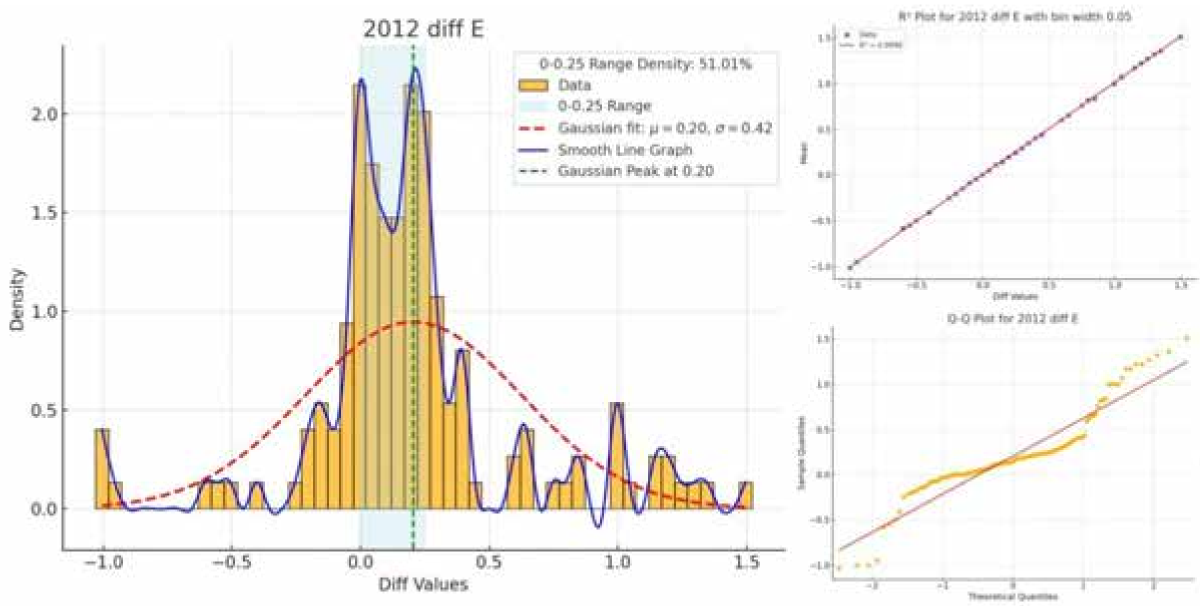


Figure S21. Statistical Analysis of Sub-Fractal Difference Values for the Izmir Plan of Metropolitan Municipality, 2012 (Eastern Region): Histogram with the Range Density, Gaussian Fit, QQ Plot and R^2 value.

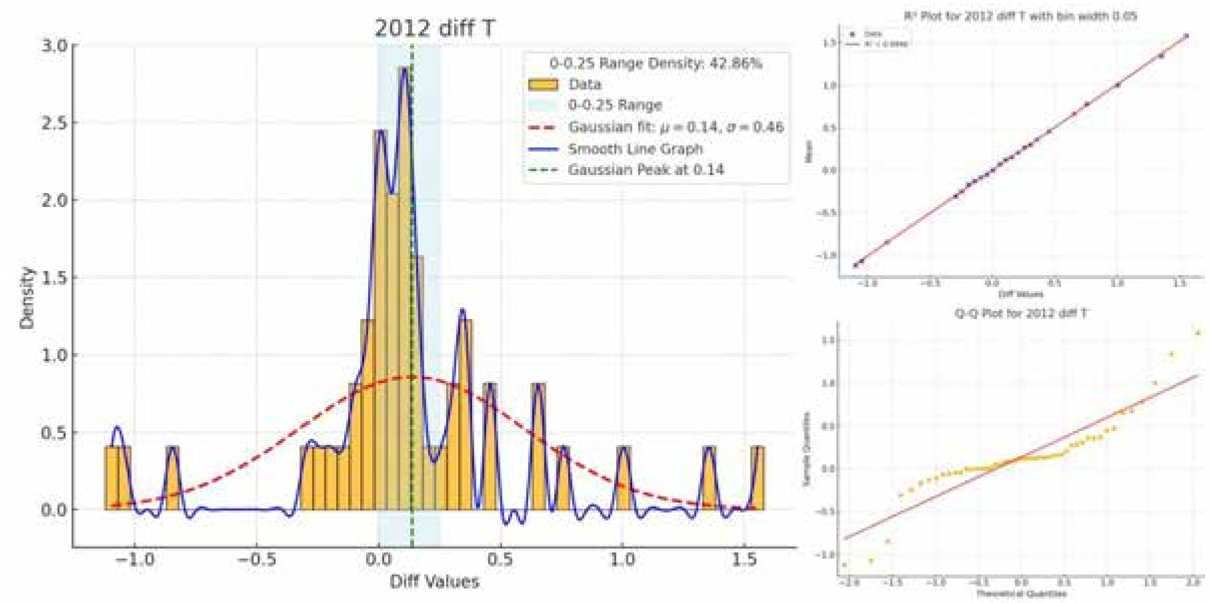


Figure S23. Statistical Analysis of Sub-Fractal Difference Values for the Izmir Plan of Metropolitan Municipality, 2012 (Central İzmir): Histogram with the Range Density, Gaussian Fit, QQ Plot and R^2 value.